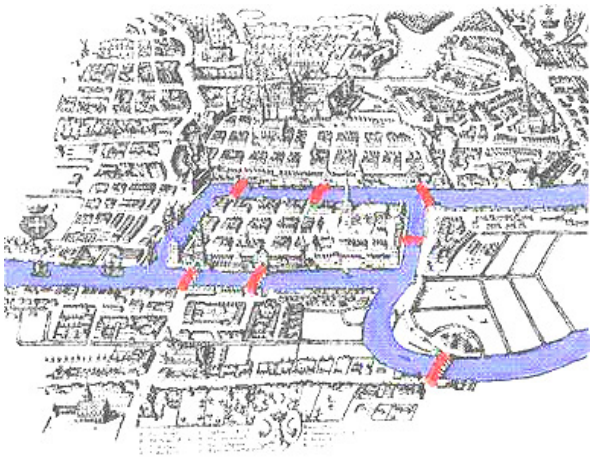
THE PROBLEMS

The Bridges of Königsberg

In the 18th century, Königsberg was a German town in which it became a traditional activity to try to go for a walk where you crossed each bridge exactly once. You could start and end at any point of your choosing.



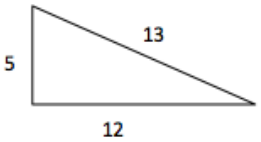
Leonhard Euler solved this problem, publishing his results in 1735. In doing so, he laid the foundations for a new area of Mathematics which we now call Graph Theory.

What was the resolution of the Königsberg Bridge problem, and can you explain it?

Finding all Pythagorean triples

Pythagoras' theorem is perhaps one of the most famous mathematical statements of all time.

A right angle triangle where all three sides lengths are integers (whole numbers) results in a Pythagorean triple. For example, the triangle below represents the Pythagorean triple (5, 12, 13).



The fact that the triangle is right angled is shown by the fact that 5, 12 and 13 fit the Pythagorean relationship:

$$5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

How many Pythagorean triples are there?

What is a primitive Pythagorean triple, and how many of these are there?

Is it possible to find all of the primitive Pythagorean triples?

Pascal's problem of points

Blaise Pascal and Pierre de Fermat are two of the most brilliant mathematicians ever to have lived. They lived in the 17th century, in France, and were friends.

The problem concerns two people, let's say Pascal and Fermat, who are playing a very simple game. The players flip a coin repeatedly; if it comes up heads, Pascal gains a point. If it comes up tails, Fermat gains a point. The first to get to 10 points wins. Both players have put £50 into a prize-pot, and the winner of the game will take the £100 prize.

Fermat is winning by 8 points to 7 when, receiving an urgent message, he suddenly has to leave the game. Pascal is left with the problem of working out what to do with the £100 prize money: how can he divide it fairly between them to reflect how close each player was to winning?

Pascal and Fermat solved this problem together, communicating by sending letters to each other. In doing so, they laid the foundations for a new area of Mathematics, what we call probability.

What was their solution and why was it fair?

Bezout's buckets and ponds

In my garden I have two buckets, one of which holds 8 litres when full, and the other of which holds 13 litres when full. I have a large quantity of water in the water butt in my garden which can provide me with as much water as I like. I want to add exactly 2 litres of water to my pond.

The buckets are oddly shaped, and have no markings on the side, so I can't part-fill them in any way accurately. Can you find and describe a way solving my problem?

Bezout was an 18th century French mathematician who solved this problem in the general case, by working out when you could and when you couldn't solve the problem for two buckets of any size and for any amount of water you want to add to the pond. Can you explain how he did this?

Egyptian fractions

The ancient Egyptians only used fractions of the form $\frac{1}{n}$, which we now call unit fractions. Any other fraction therefore had to be represented as the sum of such unit fractions, for example:

$$\frac{6}{7} = \frac{1}{2} + \frac{1}{3} + \frac{1}{42}$$

Does every fraction have a representation in Egyptian fractions?

This problem was first solved by the mathematician Leonardo Fibonacci (who the Fibonacci numbers are named after) in his book *Liber Abaci* produced in 1202.