Modelling Space-Time Curvature
Using Einsteins Field Equations

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Abstract
The aim of this project is to create a computer program that can model the curvature of space-time by utilizing the Einstein Field Equations.

Introduction

The Einstein Field equations are a set of ten equations that describe how the curvature of space time, by energy and mass, influences the interactions of gravity. To model Space-Time curvature using Einsteins field equations it is first of all important to know what each part of the equation represents both conceptually and mathematically.Before beginning if not already known, a tensor is a higher order vector description of an object, utilizing two or more vectors. Einstein derived the equations purely on intellectual and mathematic grounds as opposed to experimental data, which makes it difficult to find solutions to[1].In 1846 after Mercury's discovery it became apparent that according to Newtons theory of gravitation there had to be additional mass nearer to the sun to explain the advance of Mercuries orbit, in which no such mass was discovered[**?**]. Then later Einstein used a linearized form of his equations to calculate the precession of mercury and the deflection of light, which the precession was known then and a few years later Eddington measured the deflection of light and it matched Einsteins predictions thus he knew that linearized general relativity was correct. A year later in 1916 Karl Schwarzchild provided an analytic solution to the equations modeling a spherically symmetric object.

To model the Space-Time curvature I hope to find the tangents of the differences between the curvature in Einsteins equations and Euclidians flat space time when an object of a known radius falls into a body of a know Schwarzchild radius, to produce a graph which can be extrapolated around a 360 degree point to produce a model of Space-Time curvature. I will utilize Eulearian co-ordinates which are the co-ordinates defined by an observer at infinity which are the co-ordinate time and co-ordinate distance. I will use Stationary co-ordinates which I will define as the stationary observer at the object. And I will also use Lagrangian co-ordinates, which are defined by a fee-falling observer with the object.

I will use python within the Spyder software to program this, as it has all of the scientific packages pre-installed making it easier to use constants, a large quantity of matrices and a large quantity of derivatives.

Main Objectives

1. Research the theory behind the equations.
2. Understand the mathematics of the equations.
3. Program the equations part by part.
4. Put each part of the program together to model the curvature of space-time.

Theory

The Equation:

$$R_{\mu\nu} - g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}/c^4 \quad (1)$$

$R_{\mu\nu}$ = Ricci Tensor
$g_{\mu\nu}$ = Metric Tensor
R = Curvature Scalar
$\Lambda = CosmologicalConstant$
$T_{\mu\nu}$ = Stress-Energy Momentum Tensor

For the equation to model Space-Time curvature it has to be true for all observers no matter what the frame of reference. This means that for each tensor which involves two or more ways of describing an object, the observers, and combining them together to produce a description which is true for all of them. And to do this the entire equation is made of of tensors of rank 2 and above, where rank 0 is a scalar and rank 1 is a vector.[4][5]

The underlying tensor to each part of the equation is the metric tensor $g_{\mu\nu}$ which makes corrections for Pythagoras or can be seen as the alternative for pythagoras in curved space or in arbitrary geometry. A right angled triangle in Euclidean space would not strictly be a right angled triangle in curved space[4].

In the Einstein field equations the left side represents the curvature of space-time and the right describes the energy of mass per unit volume.The Einstein Tensor G can also be used to represent $R_{\mu\nu}$-$g_{\mu\nu}$R which is equal to $8\Pi T$

Results

In the program an arbitrary set of data calculates values for the length change due to curved space, according to the metric tensor using python within the Spyder software.

By creating a rectangular object with dimensions (40, 50, 60). Then by assigning an arbitrary distance for which the object travels in the 3 spacial dimensions (10, 20, 30) for the X observer, and (50, 75, 90) for the Y observer. Then varying the quantity of time dilation in each of the spacial dimensions between the two observers in an essence for the time dilation due to gravitational acceleration. Then graphs showing how the change in length for each dimension of the object changes due to this dilation.

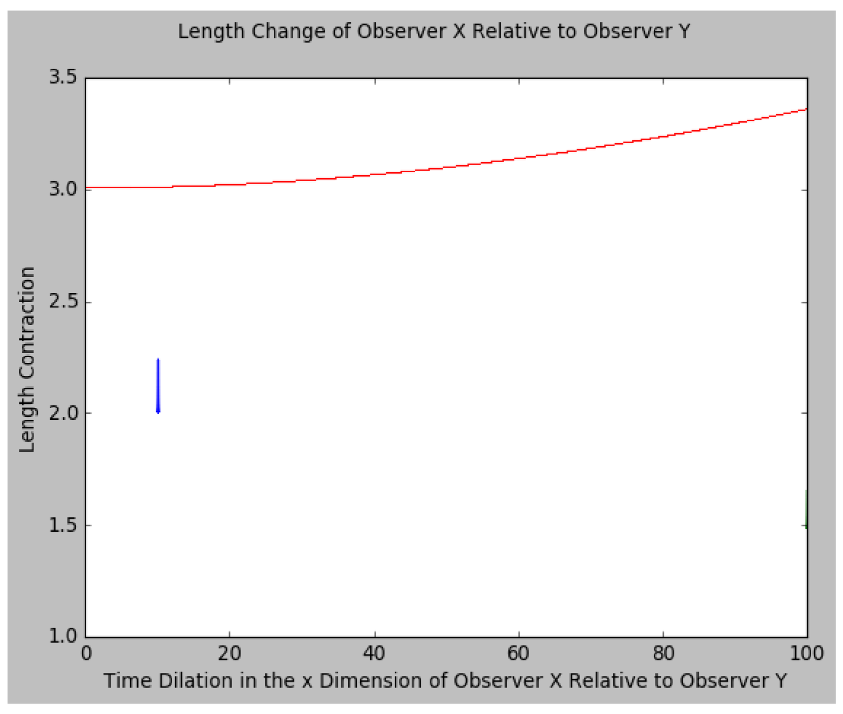

Figure 1: Length Change as Time Dilates in the x Dimension

It is clear by the graphs that as the time dilates in the one dimension its length contraction is initially exponentially proportional then when the dilation approaches 200 the relation becomes directly proportional. The other two dimensions contracts relative to the amount that the changing dimension dilates which is as expected. When the time dilation is fixed to an arbitrary (0, 10, 100) and the distance in each dimension is varied, the relationship of length contraction to distance travelled is similar to that of time dilation to length contraction. With the exception of the x dimension in which there is no variation in length contraction due to no difference in the time dilation

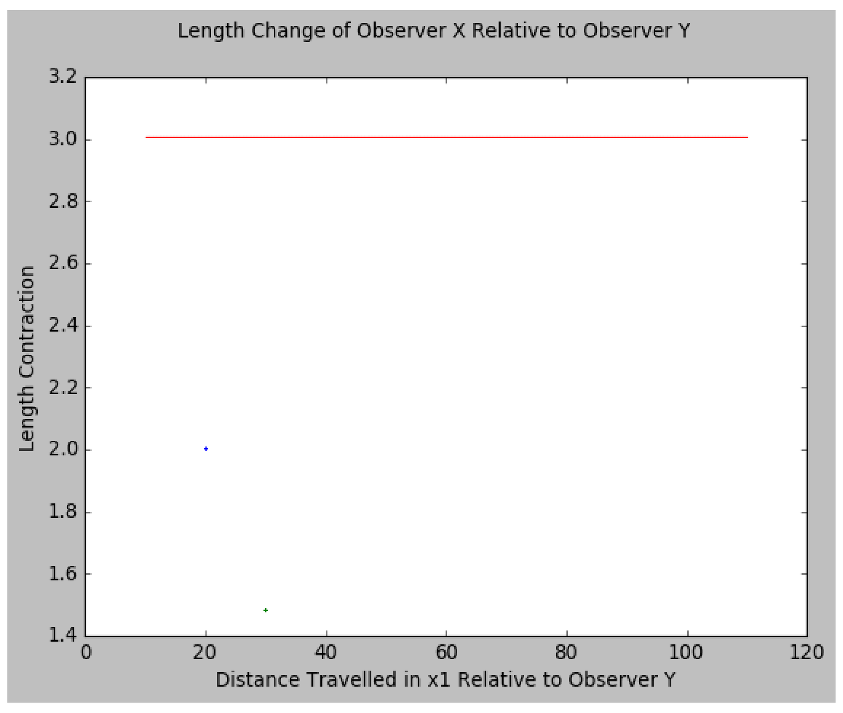

Figure 2: Length Change as the object moves through the x Dimension

Conclusions

Due to quantity of time spent on researching theory, and due to a lack of programming experience and numerous failed programs only a limited amount of results have been produced. If provided with more time, then creating a program for a Christofell symbol, which would've been using in the Ricci Tensor, could've been possible. Overall the end goal of a full Einstein field equation program is unlikely due to the lack of program experience and due to the complexity of how the energy of mass relates to the curvature of space and understanding how numerical values for each part can be tested if a program was fully complete.

References

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